



Australian
National
University

Venue

Student Number

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Research School of Economics EXAMINATION

Semester 1 - End of Semester Deferred/Supplementary, 2024

EMET7001_Semester 1 Mathematical Techniques for Economic Analysis

This paper is for ANU students.

Examination Duration:	180 minutes
Reading Time:	15 minutes
Exam Conditions:	N/A
Materials Permitted In The Exam Venue:	No materials permitted (No electronic aids are permitted e.g. laptops, phones)
Materials To Be Supplied To Students:	1 x 20 page
Instructions To Students:	See next page

INSTRUCTIONS TO STUDENTS

- Read the questions carefully.
- Questions are worth different amount of marks given in parenthesis. Sub-questions in each questions are of equal value.
- To maximize your marks, explain all the steps in your arguments.
- If any part of the question seems missing or ambiguous, state clearly the way you interpret it, and carry on with your answer.
- In solving the questions, you can use any fact from the lecture materials without proof, unless specifically asked to give details. In either case, you should clearly state the relevant fact.
- You do not need to do the questions in order, as long as you clearly mark in your answer sheets which question you are addressing.
- Do not forget to put your correct ANU uID number on the front page of the answer book.

QUESTIONS

Question 1. (10 marks)

Prove that the following properties hold for all sets A , B , C and D , and give an example to illustrate one of them.

$$A \times (B \cap C) = (A \times B) \cap (A \times C) \quad (1)$$

$$(B \cup C) \times D = (B \times D) \cup (C \times D) \quad (2)$$

Question 2. (20 marks)

Using β - δ argument, show that if a function $f: \mathbb{R} \rightarrow \mathbb{R}_{++}$ is continuous in a point a , then the function $g(x) = \ln f(x)$ is also continuous at $x = a$.

Question 3. (20 marks)

Use L'Hopital's rule to find the following limits:

1.

$$\lim_{x \rightarrow \infty} \frac{x}{e^x}$$

2.

$$\lim_{x \rightarrow \infty} \frac{x^n}{e^{nx}}, \quad n \in \mathbb{N}$$

3.

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$$

4.

$$\lim_{x \rightarrow \infty} \frac{e^{-x} + \sqrt{x} - 1}{3x + 7\sqrt{x} + 9 - e^{-7x}}$$

5.

$$\lim_{x \rightarrow -\infty} \frac{-8x^5 + 4x^4 - 3x^3 - 2x^2 + x - 5 + e^{7x+2}}{-2x^5 - 3x^3 - 2x^2 + 1}$$

Question 4. (20 marks)

Write down the Taylor expansion of the function $f(x) = e^x - \ln(x)$ around the point $a = 1$. Compute the value of this series at $x = 2$ and verify that it is equal to $f(2)$.

Hint: $\sum_{n=1}^{\infty} \frac{1}{n!} = e - 1$, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \ln 2$.

Question 5. (10 marks)

Solve the following equation in x

$$\log_x 2 - \log_2 2x = 1$$

Hint: $\log_a b = c \iff a^c = b \iff c \ln a = \ln b$

Question 6. (10 marks)

Using Gauss-Jordan elimination find the inverse of the following matrix

$$\begin{pmatrix} 0 & -1 & -2 & 1 \\ 1 & 2 & 7 & -3 \\ -3 & 2 & -4 & 8 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

Question 7. (10 marks)

Compute the following derivative with a parameter $\lambda > 0$

$$\frac{d}{dt} \int_{-t}^{\lambda t} \frac{t dx}{x}$$

END OF EXAMINATION